Restrictions on neutrino oscillations from BBN. Non-resonant case.

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Abstract

New nucleosynthesis bounds on the oscillation parameters of active neutrinos mixed with a sterile one are derived for the non-resonant case. The controversy in the literature whether to use the annihilation rate or the total reaction rate for the estimates of sterile neutrino production is resolved in favor of the annihilation rate. In contrast to previous papers, the restrictions on oscillations of electronic neutrinos are weaker than those of muonic and tauonic ones.

Neutrino oscillations in the early universe, especially if active neutrinos are mixed with sterile ones, would have a noticeable impact on primordial nucleosynthesis [1] and this permits to obtain interesting restrictions on the oscillation parameters. The results very much depend upon a possible MSW resonance transition [2] in the primeval plasma. In the case of resonance ($\delta m^2 < 0$) the oscillations are much more efficient, neutrino spectrum can be strongly distorted, and the lepton asymmetry in the sector of active neutrinos can be enhanced by several orders of magnitude. However the calculations in this case are very complicated and controversial conclusions have been reached by different groups. For the discussion and the list of references see the recent papers [3, 4]. In the non-resonant case the calculations are much simpler but there is also a disagreement between the papers [5] and all the subsequent works, see e.g. [6]-[8]. In the first papers it was assumed that the probability of production of sterile neutrinos, ν_s, is proportional to the rate of (inverse) annihilation of active neutrinos into light fermions in the plasma, γ_{ann}, while in all other papers it was argued that

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the rate of production of ν_s is proportional to a much larger total scattering rate, $\gamma_{tot} = \gamma_{el} + \gamma_{ann}$. Correspondingly the BBN (big bang nucleosynthesis) bounds on the oscillation parameters would be much more restrictive.

It is shown here that these arguments are not correct and the probability of ν_s production is indeed proportional to the annihilation rate in agreement with ref. [5], though more precise calculations presented below result in stronger bounds than found in the papers [5] but still weaker than those obtained in the above quoted papers [6]-[8], where the probability of production was taken to be proportional to γ_{tot}. The simple argument showing that the effect of oscillations vanishes in the limit of $\gamma_{ann} = 0$ is that in this limit the total number density of active and sterile neutrinos is conserved, $n_a + n_s = const$, (see below eqs. (7,8)) and roughly speaking their total energy density remains the same as in the absence of oscillations. In fact the situation is somewhat more complicated because at an early hot stage the equilibrium with respect to annihilation was also reached, so the obtained expressions do not permit to take the limit of vanishing γ_{ann}.

The difference between the approach of the present paper and all the other ones, where the BBN bounds have been derived, is that in those papers the breaking of coherence of the oscillations was described by the simplified anzats to the r.h.s. of the kinetic equations::

$$-\gamma \{g^2, \rho - \rho^{(eq)}\} \tag{1}$$

where curly brackets denote anti-commutator and g is the interaction matrix. In flavor basis it has the only nonzero entry in (a,a)-position:

$$g = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \tag{2}$$

The coefficient γ is determined by the rates of all neutrino reactions in the plasma; the calculations can be found in refs. [9, 7, 10, 3]. However this expression is not satisfactory for our purpose, in particular, because it does not conserve particle number in the case of elastic scattering. To this end we need the exact equation for the

density matrix of oscillating neutrinos derived in refs. [1, 11]. However the Fermi blocking factors will be neglected in what follows. The contribution to the r.h.s. of kinetic equations from elastic scattering of neutrinos on some other leptons, l, in the plasma is given by the anticommutators:

$$\left(\frac{d\rho(p_1)}{dt}\right)_{el} = -(A_{el}^2/2)\left(f_l(p_2)\{g^2, \rho(p_1)\} - f_l(p_4)\{g^2, \rho(p_3)\}\right) \tag{3}$$

where A_{el} is the amplitude of elastic scattering properly normalized to give a correct result for the diagonal matrix elements. It is assumed that the leptons l are not oscillating, otherwise, if the latter are oscillating neutrinos, the matrix structure of the result would be much more complicated but in the first approximation we can use the expression (3). The integration over momenta of all particles except for 1 are assumed, namely the following integration of the r.h.s. should be done:

$$\frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4\left(p_1 + p_2 - p_3 - p_4\right) \tag{4}$$

The amplitude of elastic scattering with proper symmetrization factors can be taken from tables of ref. [10].

Neutrino annihilation is described similarly:

$$\left(\frac{d\rho(p_1)}{dt}\right)_{ann} = -(A_{ann}^2/4)\left[\{g, \rho(p_1)g\bar\rho(p_2)\} + \{g, \bar\rho(p_2)g\rho(p_1)\} - f_l(p_3)f_{\bar l}(p_4)\right] \tag{5}$$

It is convenient to introduce real and imaginary parts of the non-diagonal components of neutrino density matrix:

$$\rho_{as} = \rho_{sa}^* = R + i\,I \tag{6}$$

where a and s mean respectively "active" and "sterile".

Now the kinetic equations describing evolution of density matrix of oscillating neutrinos can be written as:

$$\begin{aligned}\dot\rho_{aa}(p_1) &= -FI - A_{el}^2\left[\rho_{aa}(p_1)f_l(p_2) - \rho_{aa}(p_3)f_l(p_4)\right]\\ &\quad -A_{ann}^2\left[\rho_{aa}(p_1)\bar\rho_{aa}(p_2) - f_l(p_3)f_{\bar l}(p_4)\right],\end{aligned} \tag{7}$$

$$\begin{aligned}
\dot{\rho}_{ss}(p_1) &= FI, & (8)\\
\dot{R}(p_1) &= WI - (A_{el}^2/2)\left[R(p_1)f_l(p_2) - R(p_3)f_l(p_4)\right] \\
&\quad -(A_{ann}^2/4)\left[\rho_{aa}(p_1)\bar{R}(p_2) + \bar{\rho}_{aa}(p_2)R(p_1)\right], & (9)\\
\dot{I}(p_1) &= -WR - (F/2)\left(\rho_{ss} - \rho_{aa}\right) - (A_{el}^2/2)\left[I(p_1)f_l(p_2)\right. \\
&\quad \left. -I(p_3)f_l(p_4)\right] - (A_{ann}^2/4)\left[\rho_{aa}(p_1)\bar{I}(p_2) + \bar{\rho}_{aa}(p_2)I(p_1)\right]. & (10)
\end{aligned}$$

Here we use notations of ref. [3], so that

$$F = \delta m^2 \sin 2\theta/2E \tag{11}$$

and

$$W = \delta m^2 \cos 2\theta/2E + C_l(G_F^2 T^4 E/\alpha) \tag{12}$$

where $\alpha = 1/137$, $G_F = 1.166 \cdot 10^{-5}\text{GeV}^{-2}$, and the constant C_l depends upon the neutrino flavor, $C_e = 0.61$ and $C_{\mu,\tau} = 0.17$ [12]. We neglected the term related to charge asymmetry of the plasma, it may be a good approximation in non-resonant case.

In the cosmological FRW background the time derivative in the l.h.s. of kinetic equations goes into $d/dt \to \partial/\partial t - Hp\partial/\partial p$, where H is the Hubble parameter expressed through the thermal energy density as

$$\frac{3H^2 m_{Pl}^2}{8\pi} = \frac{\pi^2 g_*}{30} T^4 \tag{13}$$

The factor $g_* = 10.75$ is the number of relativistic species in the cosmic plasma.

We will introduce the new variables:

$$x = m_0/T \text{ and } y = p/T\,, \tag{14}$$

where the dimensional normalization factor m_0 is taken equal to 1 MeV. It is a good approximation to assume that the temperature evolves as the inverse cosmic scale factor, $T \sim 1/a(t)$. In other words, $\dot{T} = -HT$ and thus the differential operator $(\partial_t - Hp\partial_p)$ transforms into $Hx\partial_x$.

Since the oscillation rate is much faster than the reaction rate (in other words, the terms $\sim F, W$ are typically larger than the terms related to reactions) the equation (10) can be solved as

$$R = \frac{F}{2W}\,\rho_{aa} \tag{15}$$

This permits to express the imaginary part I through ρ_{aa} using eq. (9) and to substitute the result into eq. (7). If we assume that ρ_{ss} is small in comparison with ρ_{aa} so it may be neglected, the eq. (7) becomes a closed equation for a single unknown function ρ_{aa}. If elastic scattering of active neutrinos is sufficiently strong (its rate is approximately an order of magnitude larger than the rate of annihilation), then one may assume that the distribution of active neutrinos is close to kinetic equilibrium with an effective chemical potential. In other words, the anzats $\rho_{aa} = C(x)\exp(-y)$ is a good approximation. In this case both sides of eq. (7) can be integrated over d^3y so that the contribution of elastic scattering disappears and the following ordinary differential equation describing evolution of $C(x)$ is obtained:

$$\frac{dC}{dx} = -\frac{K_l}{x^4}\left[C^2 - 1 + \frac{C^2}{288}\left(6I_2 + I_1^2\right) + \frac{C}{96}\,\frac{6+16(g_L^2+g_R^2)/3}{1+2g_L^2+2g_R^2}\,\left(6I_2 - I_1^2\right)\right] \tag{16}$$

where the constant K_l is given by

$$K_l = \frac{8G_F^2\,(1+2g_L^2+2g_R^2)}{\pi^3} \tag{17}$$

and

$$I_n = \int_0^\infty dy\,y^3 e^{-y}\left(\frac{F}{W}\right)^n = \tan 2\theta \int_0^\infty \frac{dy\,y^3 e^{-y}}{(1+\beta_l\,y^2 x^{-6})^n}, \tag{18}$$

with

$$\beta_e = \frac{2.34\cdot 10^{-8}}{\delta m^2\cos 2\theta} \;\;\text{and}\;\; \beta_{\mu,\tau} = \frac{0.65\cdot 10^{-8}}{\delta m^2\cos 2\theta}, \tag{19}$$

see eqs. (11,12).

The coupling constants of neutrinos to electrons are

$$g_L^2 = (0.5\pm\sin^2\theta_W)^2 \;\;\text{and}\;\; g_R^2 = \sin^4\theta_W \tag{20}$$

The signs "±" refer to ν_e and $\nu_{\mu,\tau}$ respectively. With $\sin^2\theta_W = 0.23$ we obtain $g_L^2 + g_R^2 = 0.5858$ for ν_e and $g_L^2 + g_R^2 = 0.1258$ for $\nu_{\mu,\tau}$. Correspondingly $K_e = 0.17$ and $K_{\nu,\tau} = 0.098$. We assumed that $F/W \ll 1$ and thus the term $\sim (F/W)^2 dC/dx$ was neglected. It is a good approximation even for not very weak mixing.

Eq. (16) can be solved analytically if $|\delta| = |1 - C| \ll 1$:

$$\delta = \int_0^x dx_1 D_l(x_1) \exp\left[-\frac{2K}{3}\left(\frac{1}{x_1^3} - \frac{1}{x^3}\right)\right] \tag{21}$$

where

$$D(x)_l = K\, x^{-4}\left[a\left(6I_2 + I_1^2\right) + b_l\left(6I_2 - I_1^2\right)\right] \tag{22}$$

with $a = 3.47\cdot10^{-3}$, $b_e = 4.37\cdot10^{-2}$, and $b_{\mu,\tau} = 5.55\cdot10^{-2}$. This expression determines the freezing temperature of annihilation of active neutrinos, $T_l^{(f)} = (3/2K_l)^{1/3}$ MeV.

The increase of the total number density of oscillating neutrinos

$$\Delta n \equiv \int \frac{d^3 y}{(2\pi)^3}\Delta(\rho_{aa} + \rho_{ss}) \tag{23}$$

can be found from the sum of equations (7) and (8) and is given by

$$\left(\frac{\Delta n}{n_{eq}}\right)_l = \int_0^\infty dx\, D_l(x) \tag{24}$$

All integrals can be taken analytically, first over dx and then over dy. Finally we obtain:

$$\left(\frac{\Delta n}{n_{eq}}\right)_l = \frac{\pi K_l}{\sqrt{\beta_l}}\left(\frac{\sin 2\theta}{\cos 2\theta}\right)^2 \left[a_l + b_l + (5.14/6)(a_l - b_l)\right] \tag{25}$$

This number should be smaller than the upper limit for extra neutrino species, ΔN_ν, permitted by BBN. In particular, for electronic neutrinos we obtain (for small mixing angles):

$$\delta m^2 \sin^4 2\theta|_{\nu_e} < 5\cdot10^{-4}\Delta N_\nu^2, \tag{26}$$

while for $\nu_{\mu,\tau}$ the result is surprisingly stronger:

$$\delta m^2 \sin^4 2\theta|_{\nu_\mu,\nu_\tau} < 3.3\cdot10^{-4}\Delta N_\nu^2 \tag{27}$$

The result makes sense only if $\Delta N < 1$. Otherwise, even if the excitation of sterile neutrinos is complete, i.e. they reached equilibrium abundance, their maximum contribution to the effective number of neutrinos would be unity. The bounds (26,27) are approximately an order of magnitude weaker than those obtained in ref. [7] and an order of magnitude stronger than found in ref. [5]. In the previous papers the bounds on oscillation parameters of ν_e was stronger than those for ν_μ and ν_τ and it was related to a faster production of ν_e by charged currents. However, as we saw above, there is a competing contribution related to the refraction index of active neutrinos: it is larger for ν_e, see eqs. (12,19). Hence the oscillations of $\nu_{\mu,\tau}$ are less suppressed at high temperatures and their sterile partners are more efficiently produced.

We neglected here a distortion of neutrino spectrum by the oscillations. In contrast to the limits (26,27) that are effective even for very small mixings, because the smallness of mixing can be compensated by the efficient production of ν_s at the early stages, the distortion of spectrum of ν_e could only be developed sufficiently late, at $T < 2$ MeV and the effect could be essential for sufficiently large $\sin\theta$. One possible form of spectrum distortion is a generation of an effective chemical potential of the same sign for ν and $\bar{\nu}$, while kinetic equilibrium shape of the distribution survives. This effect dominates for sufficiently large δm^2. It was estimated in ref. [5]. A deviation of electronic neutrinos from kinetic equilibrium is essential for smaller mass difference. It was accurately calculated for $\delta m^2 \leq 10^{-7}\text{eV}^2$ in the papers [13]. Spectral distortion may have a very strong effect on primordial abundances, so the limits on the oscillation parameters could be sensible even if the data permits $\Delta N_\nu > 1$.

Electronic neutrinos are in a good thermal contact with the rest of the cosmic plasma till $T = 2$ MeV[2]. Above this temperature one may assume that the spectrum of ν_e is close to the equilibrium Fermi-Dirac form (in fact we will take Boltzmann approximation, $f_\nu = f^{(eq)} \exp(-E/T)$). Below 2 MeV ν_e may be considered as free and the impact of medium enters only through the refraction index. One can check

[2]The reaction rate is proportional to neutrino momentum p, see e.g. ref. [3] where this issue is discussed. Thus the equilibrium is maintained down to a smaller T for larger p and vice versa.

that for $\delta m^2 > 5 \cdot 10^{-6}$ eV^2 the vacuum term in effective potential (12) dominates and neutrino oscillations are close to the vacuum ones. The number density of electronic neutrinos in this case is equal to:

$$\rho_{\nu_e,\nu_e} = \left(c^4 + s^4 + 2c^2 s^2 \cos \delta m^2 t/2E\right) f^{(eq)}(E), \tag{28}$$

where $s = \sin 2\theta$ and $c = \cos 2\theta$. The last term in this expression quickly oscillates at nucleosynthesis time scale, so it can be neglected. This term would give a non-negligible contribution to neutrino spectrum for $\delta m^2 < 10^{-8}$ eV^2, but in this case the matter effects should be taken into account. From eq. (28) follows that the effective chemical potential of ν_e is

$$\xi = \mu/T = \ln\left(1 - s^2/2\right) \tag{29}$$

As is argued in ref. [5] such a distortion of equilibrium neutrino spectrum is equivalent to a renormalization of the Fermi coupling constant $G_F \to G_F[1 + \exp(\xi)]/2$. Since $\xi < 0$ the freezing of n/p-ratio would take place at higher T and more ^4He would be produced. This confines the mixing of ν_e with ν_s to the region:

$$\sin^2 2\theta < 0.32\, \Delta N_\nu \tag{30}$$

for all $\delta m^2 > 5 \cdot 10^{-6}$ eV^2.

For smaller mass differences matter effects cannot be neglected and the effective mixing angle would depend on neutrino energy:

$$\sin 2\theta_{eff} = \frac{\sin 2\theta}{1 + 1.22\, E^2 T^4 (G_F{}^2/\alpha) |\delta m^2|^{-1}} \tag{31}$$

This would modify the energy spectrum of ν_e and change the frozen value of neutron-to-proton ratio. Detailed calculations for $|\delta m^2| < 10^{-7}$ eV^2 can be found in the papers [13].

The observational limit on ΔN_ν was analyzed in a recent review [14] with the conclusion that $\Delta N_\nu < 0.2$. Thus a large mixing of ν_μ with a sterile partner proposed for explanation of the atmospheric ν_μ-deficit [15] is excluded in non-resonant case.

However, as is argued in ref. [16], one should be cautious in the data analysis and probably the safer limit is $\Delta N_\nu < 1$. If this is true an arbitrary strong mixing of active neutrinos ν_μ and ν_τ with one sterile companion would be permitted by BBN. In the case of ν_e-oscillations the distortion of ν_e spectrum could be essential and in this case an interesting bound can be observed even if $\Delta N_\nu = 1$ is permitted. However if one admits existence of one sterile neutrino, it is natural to assume that there are three of them, as e.g. in the case of Dirac-Majorana mass mixing [1]. In this case the mixing angles should be rather strongly bound by BBN, but detailed analysis with correct description of decoherence effects is necessary.

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